澳門四高校聯合入學考試（語言科及數學科）

Joint Admission Examination for Macao Four Higher Education Institutions

(Languages and Mathematics)

2024 年試題及參考答案

2024 Examination Paper and Suggested Answer

數學附加卷 **Mathematics Supplementary Paper**

1.

1.1 22

1.2

2.

3.

4.

5.

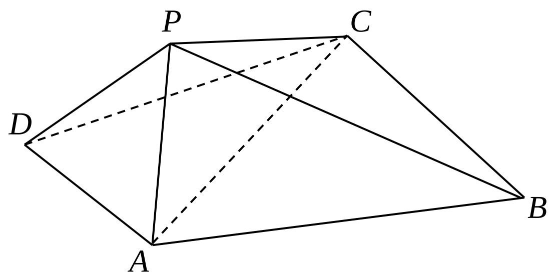
6.

7.

8.

Answer any 3 questions, each carries 20 marks. Write down the answers on the 3 lined pages following each question.

1.



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(a) (6)

(b) (6)

(c)

[:] (8)

In the above figure, is a quadrilateral, is an equilateral triangle, is a right-angled isosceles triangle with hypotenuse . –, –,

and .

(a) Show that and are perpendicular. (6 marks)

(b) Find the volume of the triangular pyramid . (6 marks)

(c) Find the cosine of the dihedral angle between plane and plane .

[Hint. Let be the mid-point of .] (8 marks)

2. (a) 1 。

(i) — , 。 (2)

(ii) 。 (4)

(iii) 。 (3)

(iv) (ii) (iii) (2)

(v) (1)

(b) k — —

1 k (8)

(a) The slant height of a right circular cone is 1 m. Suppose its base radius is m and its volume is .

(i) Show that — , . (2 marks)

(ii) Find the local maximum and local minimum values of when . (4 marks)

(iii) Find the inflection point(s) of the curve . (3 marks)

(iv) Using the results in (ii) (iii), sketch the curve (2 marks)

(v) What is the maximum possible volume of the cone? (1 mark)

(b) Let k be a positive constant. Suppose, in the first quadrant, the area of the region bounded by the line and the two curves — and — is 1. Find the value of k. (8 marks)

3. H:

(a) (2)

(b) (4)

(c) (6)

(d)

[:] (8)

Given a hyperbola H: . A non-vertical line L passing through the point interests with H at two distinct points and . Let be the slope of .

(a) Show that and satisfy the equation

. (2 marks)

(b) Find the range of (4 marks)

(c) Let be the origin. Find the value(s) of such that . (6 marks)

(d) Suppose . Find the area of the triangle .

[Hint. The segment divides the triangle into two triangles.] (8 marks)

4.

(a) (i) (4)

(ii) (4)

(b)

(8)

(c) (4)

Let .

(a) (i) Express in polar form , where and . (4 marks)

(ii) Find . Express your answer in the form , where and are real numbers. (4 marks)

(b) Let . Using theorem, show that for any positive integer ,

and .

Deduce that . (8 marks)

(c) Find the general solution of the equation . (4 marks)

5. (a) (7)

(b) k x y z :

(i) k (E) (5)

(ii) (E) (4)

(c) a

–

a (4)

(a) Factorize the determinant . (7 marks)

(b) Let k be a constant. Given the system of equations with unknowns x, y and z:

.

(i) Find the range of k such that (E) has a unique solution. (5 marks)

(ii) Suppose . Find the general solution of (E). (4 marks)

(c) Find the maximum value of a such that the system of equations

–

has a solution. For this value of a, solve the system of equations. (4 marks)

1. (a)

(1)

(2)

(b) (2)

(1) *DCB*

(c) *M* *DP*
(1)

M *DP*

M *DP*

(3)

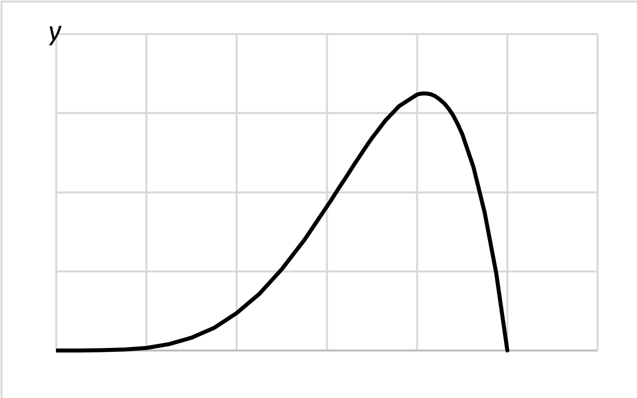
[:]

2. (a)(i)

(ii)

(iii)

(iv)



(v) $\frac{\bar{\ }}{\ }$

(b)

3. (a)

(1)

(b) (1) (1)

0

(2)

(2)

(c) (1)

(c)

5. (a)

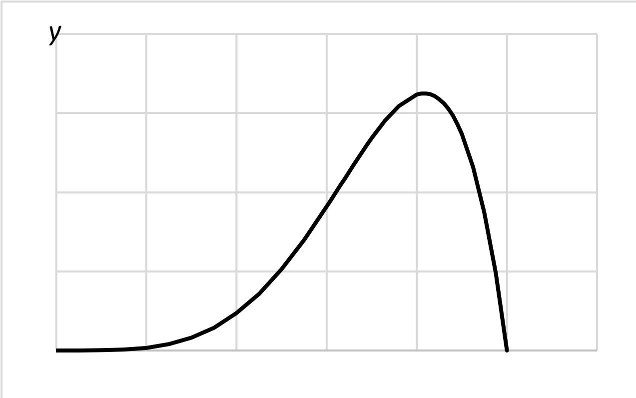
(b)(i) (E)

(ii)

(c)

3

(iv)



(v)

(b) Solving , we get or . Solving , we get or .

As the area of the bounded region is 1, we get

3. (a) , we get . That is,

. (1)

(b) As the non-vertical line and have two distinct intersection points, (1) has two distinct real roots. So, 0. That is, and

(2)

From (2), we get which is true for all *m*. Thus, the range of is .

(c) From (1), we get

Hence,

(d) Substituting into (3), we get and . So, and are positive numbers. Hence, we know that points *A* and *B* are on the same branch. We may assume and . Then,

By direct calculation,

The is

4. (a) (i)

(ii)

(b) The results follow from the sum and difference of

and .

(c)

5. (a)

(b)(i) (E) has a unique solution if and only if , that is, .

(ii) When , the system of equations becomes .

Solving , we get , .

This solution also satisfies the third equation and hence is the general solution of the system of equations.

(c) Suppose the system of equations has a solution. Then there exists such that , that is, Hence, we know that the maximum value of is 3. When , we have and the solution of the system of equations is